Making conjectures is a fundamental reasoning habit in mathematical inquiry. Geometry offers many opportunities for developing this reasoning habit through an abundance of intriguing and often surprising visual or measurable geometric relationships. Students can make conjectures by analyzing a planar or spatial configuration or by wondering whether a certain configuration can exist. Conjecturing activates their natural inquisitiveness, not only about "what might be happening" (the conjecture) but "why it should be happening" (looking for insight, validation, or refutation.) The process of seeking and making conjectures gives students the opportunity to become immersed in, and deepen their understanding of, the mathematical relationships involved, as well as to sharpen their ability to validate them. By making conjectures about novel situations, students also learn to employ mathematics in new situations, a highly desirable skill in our fast-changing world.

NCTM, Focus in High School Mathematics: Reasoning and Sense Making

# Our goal is to use varying teaching/learning strategies in order to meet the needs of all the students and the demands of the content. These strategies include, but are not limited to, the following:

Give students a new type of problem and have students arrive at solutions individually or in groups. Then share with group to collect all the different ways to solve a problem.

Present a new problem and think, pair, share.

Give students a new type of problem together with a worked out solution and have students discover and explain, in writing and verbally, how and why the solution works.

Direct instruction – Typically direct instruction will follow some exploratory time for students to play around with a new type of problem/situation/scenario. Students' brainstorming will be the start of direct instruction, with notes and examples and information that help students make sense of the new problem and place it in the context of prior knowledge.

Have students analyze a new problem: what about it looks familiar, what about it looks new, how could they start the problem or, if they can't start, what might be involved while attacking the problem. Students share ideas in writing and verbally.

Have students use technology (graphing calculators, Geometer's Sketchpad, Graphmatica, etc) to explore functions and mathematical concepts.

Have students reflect on their learning in writing and verbally. A regular class wrap up will include asking students to write what they learned in the day's work, what questions they still have, what it reminds them of from past work, and other associations they have with the new material.

Expose students to complex problems that involve many concepts and lend themselves to a variety of solutions and strategies. These could be problems that take anywhere from 15 minutes to an hour to multiple days to solve. **Instructional goals** 

Nurture an appreciation for the distinct nature of mathematics as an abstract language system that is internally consistent and understood through rigorous analytical thinking skills.

Nurture an appreciation for how the analytical thinking and problem solving skills honed in mathematics is essential for students' current and future lives regardless of whether they choose a mathematical or scientific field.

Wherever possible, tie the mathematical content to other fields such as economics, literature, all the sciences, psychology, politics, etc., so that students can see the relevance and use of mathematics in other contexts.

Nurture numeracy and statistical savvy so that students may be critical consumers of statistical information in their current and future lives.

A constant goal is to achieve depth of understanding and connection, despite a much too full list of topics prescribed by the State of New York.

Nurture mathematical reasoning and analytical skills and the ways to express one's reasoning, both verbally and in writing. We want to encourage students to look for and recognize patterns, internal structure, regularities or irregularities both in "real-world" problems and in the symbolic language of mathematics. We want students to see when patterns are meaningful as opposed to when they are by chance or accidental. We want students to justify their solutions and to see why those solutions make sense.

#### Assessment

We plan to use both formal and informal assessments to ascertain understanding. Assessments will also be both formative and summative.

Projects – research and writing projects, statistics projects that involve gathering and analyzing data, solving and explaining solutions to complex, multi-faceted problems

Tests and quizzes

Group work – group work allows the teacher to circulate and listen in, thus giving the teacher an idea of student understanding and misconceptions.

Written descriptions of solutions to problems – students will be asked to describe their process for solving a particular problem in writing, which will give the teacher an insight into student understanding of the method being assessed.

#### Homework

We hope to train students to make homework a productive, reflective process. Homework is a time to practice problem solving skills and thinking processes. By providing solutions, we hope to encourage students to check their own work and work independently to find their own mistakes and identify any misunderstandings or gaps in knowledge.

#### **Topics for Geometry**

Unit 1: Reasoning Unit 2: Coordinate Geometry Unit 3: Transformational Geometry Unit 4: Constructions Unit 5: Locus Unit 5: Locus Unit 6: Triangle Congruence Unit 7: Triangle Congruence/Inequality Unit 8: Polygons Unit 9: Similar Triangles Unit 10: Circles

Unit 11: 3-D Solids

# **Unit 1: Reasoning**

- 1. How does formal logic help you make decisions?
- 2. Why is it important to justify all the steps in the process of reasoning?

Time	Perform Ind	Content	Lessons	Vocabulary
Sept. (3 wks)	G.G.24	Determine the negation of a statement and establish its truth value	<ul> <li>1: Statements</li> <li>Negations and truth values</li> <li>Conjunction/Disjunction</li> </ul>	Statement Conjunction Disjunction
	G.G.25	Know and apply the conditions under which a compound statement (conjunction, disjunction, conditional, biconditional) is true.	2: Compound Statements • Conditionals • Biconditionals • Inverse • Converse • Converse • Contrapositive • Logical Equivalence 3: Applications • Solve problems using formal and informal logic 4: Assessment	Conditional Biconditional Inverse Converse Contrapositive Logical Equivalence
	G.G.26	Identify and write the inverse, converse, and contrapositive of a given conditional statement and note the logical equivalences		

# **Unit 2: Coordinate Geometry**

- 1. What is the relationship between cartography and coordinate geometry?
- 2. How can mathematical formulas be used to validate properties of polygons?

Time	Perform Ind	Content	Lessons	Vocabulary
Sept-Oct (4-5 wks)	G.G.62	Find the slope of a perpendicular line, given the equation of a line	1: Investigate Software • Parallel • Perpendicular • Slope	Parallel Perpendicular
	G.G.63	Determine whether two lines are parallel, perpendicular, or neither, given their equations	2: Investigate • Distance-Length • Midpoint	Line segment Midpoint Distance
	G.G.65	Find the equation of a line, given a point on the line and the equation of a line parallel to the desired line	<ul> <li>3: Writing Equations</li> <li>Parallel/Perpendicular</li> <li>Graphing Solution of Quadratic/Linear System</li> </ul>	Ordinate Abscissa
	G.G.40	Find the equation of a line given a point on the line and the equation of a line perpendicular to the given line	<ul><li>4: Informal Proofs</li><li>Triangles</li><li>Software/Applications</li></ul>	Isosceles Equilateral Scalene Right
	G.G.40	Investigate, justify, and apply theorems about trapezoids involving their angles, sides, medians, and diagonals	5: Polygon Properties • Software Applications • Parallelogram • Rectangle • Rhombus • Square • Trapezoid 6: Polygon Properties • Algebra Applications	Parallelogram Rectangle Rhombus Square Trapezoid
	G.G.66	Find the midpoint of a line segment, given its endpoints	7-8: Informal proofs ● Polygons	
	G.G.67	Find the length of a line segment, given its endpoints	9: Applications 10:Assessment	
	G.G.68	Find the equation of a line that is the perpendicular bisector of a line segment, given the endpoints of the line segment		
	G.G69	Investigate, justify, and apply the properties of triangles and quadrilaterals in the coordinate plane, using the distance, midpoint, and slope formulas		

# **Unit 3: Transformational Geometry**

- 1. What are the similarities and differences among transformations?
- 2. How are the principles of transformational geometry used in art, architecture and fashion?
- 3. What are the applications of transformations?
- 4. How are algebraic and geometric transformations related?

Time	Perform Ind	Content	Lessons	Vocabulary
Oct-Nov (4 weeks)	G.G.54	Define, investigate, justify, and apply isometries in the plane	<ul> <li>1: Reflection</li> <li>Symbolic Notation</li> <li>Origin x=0, y=0, y=x</li> </ul>	Image Pre-image Symmetry Reflection
	G.G.55	Investigate, justify, and apply the properties that remain invariant under translations, rotations, reflections, and glide reflections	2: Rotation • Symbol Notation • Origin • 90 degree, 180 degree	Rotation clockwise Counter-clockwise
	G.G.56	Identify specific isometries by observing orientation, numbers of invariant points, and/or parallelism	3: Translations <ul> <li>Symbolic Notations</li> </ul>	Translation
	G.G.57	Justify geometric relationships using transformational techniques	<ul> <li>4: Dilation/Similarities</li> <li>Symbolic Notations</li> <li>Origin</li> <li>Assessment</li> </ul>	Dilation Similarity
	G.G.58	Define, investigate, justify and apply similarities		
	G.G.59	Investigate, justify, and apply the properties that remain invariant under similarities	5: Glide Reflections <ul> <li>Symbolic Notations</li> </ul>	Glide Reflection
	G.G.60	Identify specific similarities by observing orientation, numbers of invariant points, and/or parallelism	6-7: Isometries ● Definitions ○ Direct ○ Opposite ● Investigate all Transformations	
	G.G.61	Investigate, justify, and apply the analytical representations for translations, rotations about the origin of 90° and 180°, reflections over the lines $x = 0$ , $y = 0$ , and $y = x$ , and dilations centered at the origin	8: Applications	
			9: Assessment Note: Investigate software used throughout lessons	

# **Unit 4: Constructions**

- 1. What geometric conclusions can be drawn from using constructions as your hypotheses?
- 2. What occupations may use the geometric principles of constructions?

Time	Perform Ind	Content	Lessons	Vocabulary
Nov. (2 wks)	G.G.17	Construct a bisector of a given angle using a straightedge and compass, and justify the construction	1: Basic Skills • Segments • Angles • Triangles	Construction Straightedge Compass Point Arc Isosceles Scalene Equilateral
	G.G.18	Construct the perpendicular bisector of a given segment, using a straightedge and compass, and justify the construction	2: Bisecting Skills • Segments • Angles • Perpendicular Bisectors	Bisector Equivalent Perpendicular
	G.G.19	Construct lines parallel (or perpendicular) to a given line through a given point, using a straightedge and compass, and justify the construction	3: Applications <ul> <li>Parallel Lines</li> </ul>	Parallel
	G.G.20	Construct an equilateral triangle, using a straightedge and compass, and justify the construction	4: Assessment (Project)	

#### Unit 5: Locus

- 1. How are locus and constructions related?
- 2. How dos locus lead to improving your ability to follow or give directions?

Time	Perform Ind	Content	Lessons	Vocabulary
Dec (3 wks)	G.G.21	Investigate and apply the concurrence of medians, altitudes, angle bisectors, and perpendicular bisectors of triangles	<ul> <li>1: 5 Basic Theorems</li> <li>Investigation software</li> </ul>	Locus Loci Radius Compound loci Angle bisectors Perpendicular bisector
	G.G.22	Solve problems using compound loci	<ul> <li>2: Compound Locus</li> <li>Investigation software</li> <li>Real-life situations</li> <li>Coordinate plane</li> </ul>	
	G.G.23	Graph and solve compound loci in the coordinate plane	<ul> <li>3: Compound Locus</li> <li>Write Equations</li> <li>Circle Equations</li> <li>Graphing Solution of Quadratic/Linear System</li> </ul>	Coordinate plane
	G.G.71	Write the equation of a circle, given its center and radius or given the endpoints of a diameter	4: Concurrence in Triangles • Investigation software • Median • Altitudes	Median Altitude
	G.G.72	Write the equation of a circle given its graph (center is an ordered pair of integers an the radius is an integer)	Angle bisector     Perpendicular bisector	
	G.G.73	Find the center and radius of a circle, given the equation of the circle in center-radius form	5: Applications of constructions • Word problems 6: Assessment	
	G.G.74	Graph circles of the form $(x - h)^2 + (j - k)^2 = r^2$		
	G.G.70	Solve systems of equations involving one linear equation and one quadratic equation graphically		

# Unit 6: Euclidean Proofs: Informal/Formal Triangle Congruence

- 1. How are the properties, postulates and theorems used in proofs and mathematics?
- 2. How do civil engineers use knowledge of triangle properties?

Time	Perform Ind	Content	Lessons	Vocabulary
Dec-Feb. (5 wks)	G.G.27	Write a proof arguing from a given hypothesis to a given conclusion	<ol> <li>Proof Argument</li> <li>Hypothesis</li> <li>Conclusion</li> <li>Pythagorean Theorem</li> </ol>	Angles Hypothesis Postulates Conclusion Inverse Converse
	G.G.30	Investigate, justify, and apply theorems about the sum of the measures of the angles of a triangle	<ul> <li>2-3: Properties of Triangles</li> <li>Investigative software</li> <li>Sum of angles</li> <li>Side/angle relationship</li> </ul>	Pythagorean Exterior angle
	G.G.33	Investigate, justify, and apply the triangle inequality theorem	<ul><li>Exterior angles</li><li>Triangle inequalities</li></ul>	
	G.G.34	Determine either the longest side of a triangle given the three angle measures or the largest angle given the lengths of three sides of a triangle	<ul> <li>4: Triangle congruence – Investigate Theorems</li> <li>SSS</li> <li>ASA</li> <li>AAS</li> <li>SAS</li> </ul>	
	G.G.28	Determine the congruence of two triangles by using one of the five congruence techniques (SSS, SAS, ASA, AAS, HL) given sufficient information about the sides and/or angles of two congruent angles	<ul> <li>5: Theorem Decisions</li> <li>Definitions</li> <li>Fill-in the blank proofs</li> <li>6: Theorem Decisions</li> <li>Postulates</li> <li>Fill-in the blank proofs</li> </ul>	Postulate Reflexive
	G.G.35	Determine if two lines cut by a transversal are parallel, based on the measure of given pairs of angles formed by the transversal and the lines	<ul> <li>7-8: Triangle Congruence Proofs</li> <li>Hypothesis to Conclusion</li> <li>Apply all Theorems</li> </ul>	
	G.G.48	Investigate, justify, and apply the Pythagorean theorem and its converse	<ul> <li>9: Applications</li> <li>More triangle proofs</li> <li>10: Assessment: Midterm Exam</li> </ul>	

# Unit 7: Euclidean Proofs: Informal/Formal Triangle Congruence and Inequality

# **Essential Questions:**

1. What is the relationship between congruence and inequality

Time	Perform Ind	Content	Lessons	Vocabulary
Feb-Mar (4 wks)	G.G.30	Investigate, justify, and apply theorems about the sum of the measures of the angles of a triangle	Triangle congruence         1: CPCTC Proofs         2: Other Triangle Theorems         • HL Theorem         • Isosceles/Converse	Corresponding parts Hypotenuse Leg Isosceles Converse Base angles
	G.G.31	Investigate, justify, and apply the isosceles triangle theorem and its converse	3: Overlapping Triangles <ul> <li>Apply theorems</li> </ul>	
	G.G.32	Investigate, justify, and apply theorems about geometric inequalities, using the exterior angle theorem	Triangle inequalities 4: Inequality postulates • Proofs	Altitude Median Adjacent Complementary
	G.G.29	Identify corresponding parts of congruent triangles	5: Exterior angle theorem • Proofs 6-7: Applications	Supplementary Exterior angle
			8: Assessment	

# Unit 8: Euclidean Proofs: Informal/Formal Polygons

- 1. What are the unique properties and characteristics associated with geometric figures?
- 2. How are the properties of polygons used in art, music, and engineering?

Time	Perform Ind	Content	Lessons	Vocabulary
Mar (3 wks)	G.G. 36	Investigate, justify, and apply theorems about the sum of the measures of the interior and exterior angles of polygons	<ol> <li>Properties of Polygons</li> <li>Identify types of polygons</li> <li>Investigate parallel lines cut by a transversal</li> <li>Sum of measures of interior and exterior angles</li> </ol>	Regular polygons Transversal Alternate exterior angles Alternate interior angles Corresponding angles
	G.G. 37	Investigate, justify, and apply theorems about each interior and exterior angle measure of regular polygons	<ul> <li>2: Parallelogram proofs</li> <li>Basic proofs</li> <li>Rectangle, Rhombus, Square</li> </ul>	Rhombus Rectangles Parallelogram Square
	G.G. 38	Investigate, justify, and apply theorems about parallelograms involving their angles, sides, and diagonals	<ul><li>3: Parallelogram Proof</li><li>Application proofs</li></ul>	
	G.G.39	Investigate, justify, and apply theorems about special parallelograms involving their angles, sides, and diagonals	4: trapezoid Proofs <ul> <li>Median</li> <li>Diagonals</li> <li>Isosceles Trapezoid</li> </ul>	Trapezoid Diagonals Isosceles
	G.G.40	Investigate, justify, and apply theorems about trapezoids involving their angles, sides, medians, and diagonals	5: Applications	
	G.G.41	Justify that some quadrilaterals are parallelograms, rhombuses, rectangles, squares, or trapezoids	6: Assessment	

# Unit 9: Euclidean Proofs: Informal/Formal Similarity of Triangles and its Applications

- 1. What are the properties and theorems that connect multiple geometry figures (e.g. congruence, similarity, etc) to real world problems?
- 2. How can similarity foster conclusions about mean proportionality?

Time	Perform Ind	Content	Lessons	Vocabulary
Apr (3 wks)	G.G. 44	Establish similarity of triangles using the following theorems: AA, SAS, and SSS	<ol> <li>1: Investigate Theorems</li> <li>AA</li> <li>Basic similar proofs</li> </ol>	Similarity
	G.G.45	Investigate, justify, and apply theorems about similar triangles	<ul> <li>2: Similarities of triangles</li> <li>Sides are in proportion</li> <li>Proportion definition</li> </ul>	Proportional
	G.G.42	Investigate, justify, and apply theorems about geometric relationships, based on the properties of the line segment joining the midpoints of two sides of the triangle	3: Median/Centroid theorems <ul> <li>Investigate theorems</li> <li>Proofs</li> </ul>	Median Centroid
	G.G.43	Investigate, justify, and apply theorems about the centroid of a triangle, dividing each median into segments whose lengths are in the ratio 2:1	<ul> <li>4-5: Mean proportionality</li> <li>Investigate/special right triangles</li> <li>Proofs</li> </ul>	Altitude Hypotenuse
	G.G.46	Investigate, justify, and apply theorems about proportional relationships among the segments of the sides of the triangle, given one or more lines of the sides of the triangle, given one or more lines parallel to one side of a triangle and intersecting the other two sides of the triangle	6: Applications 7: Assessment	
	G.G.47	<ul><li>Investigate, justify, and apply theorems about mean proportionality:</li><li>altitude to the hypotenuse of a right triangle</li></ul>		

# Unit 10: Euclidean Proofs: Informal/Formal Circles

- 1. How can angle and segment theorems of circles be directly applied to real world applications?
- 2. How are the similarity triangle theorems applied to proofs about circles?

Time	Perform Ind	Content	Lessons	Vocabulary
Apr-May (4 wks)	G.G. 49	Investigate, justify and apply theorems regarding chords of a circle	<ol> <li>Arcs and Angles</li> <li>Basic vocabulary</li> <li>Central angles</li> <li>Inscribed angles</li> </ol>	Radius Diameter Angle Vertex Central angles Arc
	G.G.50	Investigate, justify, and apply theorems about tangent lines to a circle	2: Chord Theorems <ul> <li>Investigate arc measures</li> <li>Parallel chord theorems</li> </ul>	Minor arc Major arc Chord
	G.G.51	Investigate, justify, and apply theorems about the arcs determined by the rays of angles formed by two lines intersecting a circle	3: Tangents and Secants Investigate theorems Definitions Angles formed by Tangents/Chords/Secants	
	G.G.52	Investigate, justify, and apply theorems about arcs of a circle cut by two parallel lines.	<ul> <li>4-5: Measures of Tangents, Secants, chord segments</li> <li>Investigate theorems</li> <li>Area of sectors</li> </ul>	Tangent Secant
	G.G.53	Investigate, justify, and apply theorems regarding segments intersected by a circle	6: Circle proofs 7: Circle proofs 8: Applications	Sector
			9: Assessment	

#### Unit 11: 3-D Solids

- 1. How does your knowledge of Euclidean 2-D geometry theorems validate 3-D solids?
- 2. How is volume derived from area?
- 3. When and why have these shapes been used in ancient history?

Time	Perform Ind	Content	Lessons	Vocabulary
May-Jun (4 wks)	G.G.1	Know and apply that if a line is perpendicular to each of two intersecting lines at their point of intersection, then the line is perpendicular to the plane determined by them	<ol> <li>Investigate planes</li> <li>Parallel lines/Planes</li> <li>Perpendicular lines/Plaines</li> </ol>	3-D solids Lateral edges Lateral faces
	G.G.2	Know and apply that through a given point there passes one and only one plane perpendicular to a given line	<ul> <li>2: Prisms and Pyramids</li> <li>Parallel Edges (Properties)</li> <li>Volume/Altitude relations</li> </ul>	Planes Volume Altitude Prism Cube
	G.G.3	Know and apply that through a given point there passes one and only one line perpendicular to a given plane	3: Cylinder/Right Circular Cones • Properties • Theorems • Volume	Pyramid
	G.G.4	Know and apply that two lines perpendicular to the same plane are coplanar	4: Sphere • Properties • Theorems • S.A./Volume	Cylinder
	G.G.5	Know and apply that two planes are perpendicular to each other if and only if one plane contains a line perpendicular to the second plane	5: Applications	Right circular cone
	G.G.6	Know and apply that if a line is perpendicular to a plane, then any line perpendicular to the given line at its point of intersection with the given plane is in the given plane	6: Assessment: Regents exam	Sphere Surface area
	G.G.7	Know and apply that if a line is perpendicular to a plane, then every plane containing the line is perpendicular to the given plane		
	G.G.8	Know and apply that if a plane intersects two parallel planes, then the intersection is two parallel lines		
	G.G.9	Know and apply that two planes perpendicular to the same line are parallel		
	G.G.10	Know and apply that the lateral edges of a prism are congruent and parallel		

G.G.11	equal volumes if their bases have equal areas and their altitudes are equal	
G.G.12	Know and apply that the volume of a prism is the product of the area of the base and the altitude	
G.G.13	<ul> <li>Apply the properties of a regular pyramid, including:         <ul> <li>Lateral edges are congruent</li> <li>Lateral faces are congruent isosceles triangles</li> <li>Volume of a pyramid equals one-third the product of the area of the base and the altitude</li> </ul> </li> </ul>	
G.G.14	<ul> <li>Apply the properties of a cylinder, including:         <ul> <li>Bases are congruent</li> <li>Volume equals the product of the area of the base and the altitude</li> <li>Lateral area of a right circular cylinder equals the product of an altitude and the circumference of the base</li> </ul> </li> </ul>	
G.G.15		
G.G.16		